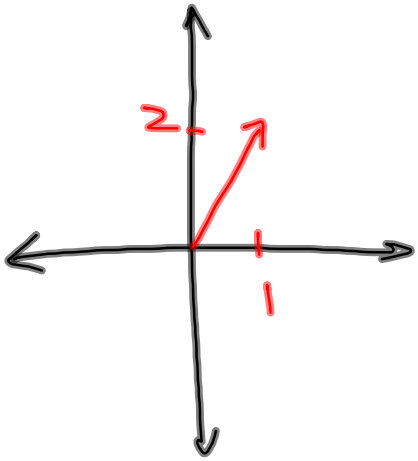
Orthogonality.

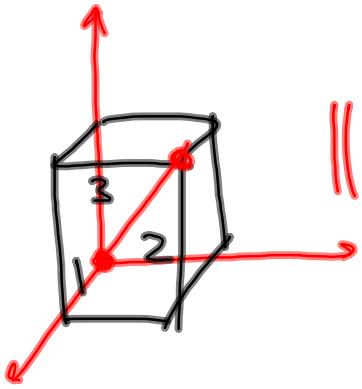
length of a vector.

$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\|x\| = \sqrt{1^2 + 2^2} = \sqrt{5}$ ← length of x



$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$



$\|x\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\|x\|^2 = 1^2 + 2^2 + 3^2$$

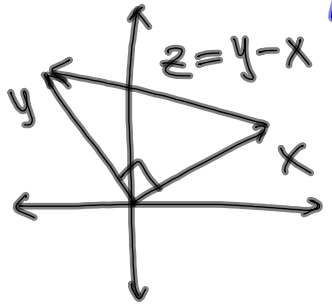
$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 1^2 + 2^2 + 3^2$$

$$x^T x = \|x\|^2$$

$$X = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

$$\|X\|^2 = X^T X = a_1^2 + a_2^2 + \cdots + a_n^2$$

$$y-x=\begin{bmatrix} b_1-a_1 \\ b_2-a_2 \end{bmatrix} \quad x=\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad y=\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$



$$\|x\|^2+\|y\|^2=\|y-x\|^2$$

$$a_1^2+a_2^2+b_1^2+b_2^2=(b_1-a_1)^2+(b_2-a_2)^2$$

$$=b_1^2-2a_1b_1+a_1^2$$

$$+b_2^2-2a_2b_2+a_2^2$$

$$0=-2a_1b_1-2a_2b_2$$

$$0=a_1b_1+a_2b_2$$

$$x+z=y$$

$$z=y-x$$

*

$$\langle a_1, a_2\rangle \cdot \langle b_1, b_2\rangle = 0$$

$$\begin{bmatrix} a_1 & a_2 \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}=0$$

$$x^T y=0$$

$$x=\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad y-x=\begin{bmatrix} b_1-a_1 \\ b_2-a_2 \\ \vdots \\ b_n-a_n \end{bmatrix}$$

$$y=\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$a_1b_1+a_2b_2+\cdots+a_nb_n=0$$

1) Let $x=\begin{bmatrix}2\\3\end{bmatrix}$ $y=\begin{bmatrix}-1\\0\end{bmatrix}$

Is x perpendicular to y? No

$$x^T y \neq 0$$

2) If x is perpendicular to y, where

$x=\begin{bmatrix}2\\3\end{bmatrix}$ $y=\begin{bmatrix}-1\\k\end{bmatrix}$, find K.

$$x^T y = 0$$

$$\begin{bmatrix}2 & 3\end{bmatrix}\begin{bmatrix}-1\\k\end{bmatrix}=0$$

$$-2+3k=0$$

$$k=\frac{2}{3}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} \begin{bmatrix} -b \\ a \end{bmatrix}$$